South Slave Divisional Education Council

Math 31

Curriculum Package

February 2012

2012

Creating Futures

Precalculus and Limits

General Learner Expectations	Specific Learner Outcomes
Students are expected to understand that functions, as well as variables, can be combined, using operations, such as addition and multiplication, and demonstrate this, by: • describing the relationship among functions after performing translations, reflections, stretches and compositions on a variety of functions • drawing the graphs of functions by applying transformations to the graphs of known functions • expressing final algebraic and trigonometric answers in a variety of equivalent forms, with the form chosen to be the most suitable form for the task at hand • constructing mathematical models for situations in a broad range of contexts, using algebraic and trigonometric functions of a single real variable.	Students will demonstrate conceptual understanding of the algebra of functions, by: • illustrating different notations that describe functions and intervals • expressing, in interval notation, the domain and range of functions • expressing the sum, product, difference and quotient, algebraically and graphically, given any two functions • expressing, algebraically and graphically, the composition of two or more functions • illustrating the solution sets for linear, quadratic and absolute value inequalities $\lvert P(x)\rvert \ge a$ $\lvert P(x)\rvert \le a$ • $ax^2+bx+c \ge d$ • illustrating the difference between the concepts of equation and identity in trigonometric contexts Students will demonstrate competence in the procedures associated with the algebra of functions, by: • using open, closed and semi-open interval notation • finding the sum, difference, product, quotient and composition of functions • solving inequalities of the types $\lvert P(x)\rvert \ge a$ $\lvert P(x)\rvert \le a$ $\left\lvert \frac{P(x)}{Q(x)}\right\rvert \ge a$ $\frac{P(x)}{Q(x)} \ge a$ $ax^2+bx+c \ge d$ • using the following trigonometric identities: • primary and reciprocal ratio • sum and difference sin (A ± B) cos (A ± B) • double and half angle • Pythagorean • to simplify expressions and solve equations, express sums and differences as products, and rewrite expressions in a variety of equivalent forms. Students will demonstrate problem-solving skills, by: • modelling problem situations, using sums, differences, products and quotients of functions • investigating the connections between the algebraic form of a function f(x) and the symmetries of its graph.

Precalculus and Limits

General Learner Expectations	Specific Learner Outcomes
Students are expected to understand that functions can be transformed, and these transformations can be represented algebraically and geometrically, and demonstrate this, by: • describing the relationship among functions after performing translations, reflections, stretches and compositions on a variety of functions • drawing the graphs of functions by applying transformations to the graphs of known functions • expressing final algebraic and trigonometric answers in a variety of equivalent forms, with the form chosen to be the most suitable form for the task at hand • constructing mathematical models for situations in a broad range of contexts, using algebraic and trigonometric functions of a single real variable.	Students will demonstrate conceptual understanding of the transformation of functions, by: • describing the similarities and differences between the graphs of $y = f(x)$ and $y = af[k(x+c)]+d$, where a, k, c and d are real numbers • describing the effects of the reflection of the graphs of algebraic and trigonometric functions across any of the lines $y = x$, $y = 0$, or $x = 0$ • describing the effects of the parameters a, b, c and d on the trigonometric function $f(x) = a \sin [b(x+c)]+d$ • describing the relationship between parallel and perpendicular lines • describing the condition for tangent, normal and secant lines to a curve • linking two problem conditions to a system of two equations for two unknowns. Students will demonstrate competence in the procedures associated with the transformation of functions, by: • sketching the graph of, and describing algebraically, the effects of any translation, reflection or dilatation on any of the following functions or their inverses: o linear, quadratic or cubic polynomial o absolute value o reciprocal o exponential o step • sketching and describing, algebraically, the effects of any combination of translation, reflection or dilatation on the following functions: o $f(x) = a \sin [b(x+c)]+d$ o $f(x) = a \cos [b(x+c)]+d$ o $f(x) = a \tan [b(x+c)]$ • finding the equation of a line, given any two conditions that serve to define it • solving systems of linear-linear, linear-quadratic or quadratic-quadratic equations. Students will demonstrate problem-solving skills. by: • translating problem conditions into equation or inequality form.
Students are expected to understand that final answers may be expressed in different equivalent forms, and demonstrate this, by: • giving examples of the differences between intuitive and rigorous proofs in the context of limits, derivatives and integrals • expressing final algebraic and trigonometric answers in a variety of equivalent forms, with the form chosen to be the most suitable form for the task at hand	Students will demonstrate conceptual understanding of equivalent forms, by: • describing what it means for two algebraic or trigonometric expressions to be equivalent. Students will demonstrate competence in the procedures associated with the construction of equivalent forms, by: • factoring expressions with integral and rational exponents, using a variety of techniques • rationalizing expressions containing a numerator or a denominator that contains a radical • simplifying rational expressions, using any of the four basic operations. Students will demonstrate problem-solving skills, by: • illustrating the difference between verification and proof in the comparison of two algebraic or trigonometric expressions.

Precalculus and Limits

General Learner Expectations	Specific Learner Outcomes
Students are expected to understand that descriptions of change require a careful definition of limit and the precise use of limit theorems, and demonstrate this, by: • identifying and giving examples of the strengths and limitations of the use of technology in the computation of limits, derivatives and integrals • giving examples of the differences between intuitive and rigorous proofs in the context of limits, derivatives and integrals • giving examples of the limits of functions and sequences, both at finite and infinite values of the independent variable • computing limits of functions, using definitions, limit theorems and calculator/computer methods.	Students will demonstrate conceptual understanding of limits and limit theorems, by: • explaining the concept of a limit • giving examples of functions with limits, with left-hand or right-hand limits, or with no limit • giving examples of bounded and unbounded functions, and of bounded functions with no limit • explaining, and giving examples of, continuous and discontinuous functions • defining the limit of an infinite sequence and an infinite series • explaining the limit theorems for sum, difference, multiple, product, quotient and power • illustrating, using suitable examples, the limit theorems for sum, difference, multiple, product, quotient and power Students will demonstrate competence in the procedures associated with limits and limit theorems, by: • determining the limit of any algebraic function as the independent variable approaches finite or infinite values for continuous and discontinuous functions • sketching continuous and discontinuous functions, using limits, intercepts and symmetry • calculating the sum of an infinite convergent geometric series • using definitions and limit theorems to determine the limit of any algebraic function as the independent variable approaches a fixed value • using definitions and limit theorems to determine the limit of any algebraic function as the independent variable approaches $\pm\infty$. Students will demonstrate problem-solving skills, by: • proving and illustrating geometrically, the following trigonometric limits: $\lim_{x\to 0}\frac{\sin x}{x}\ or\ \lim_{x\to 0}\frac{x}{\sin x}=1$ AND $\lim_{x\to 0}\frac{\cos x-1}{x}=0$ • using the basic trigonometric limits, combined with limit theorems, to determine the limits of more complex trigonometric expressions • comparing numerical and algebraic processes for the determination of algebraic and trigonometric limits

Derivatives and Derivative Theorems

General Learner Expectations	Specific Learner Outcomes
Students are expected to understand that the derivative of a function is a limit that can be found, using first principles, and demonstrate this, by: • giving examples of the differences between intuitive and rigorous proofs in the context of limits, derivatives and integrals • connecting the derivative with a particular limit, and expressing this limit in situations like secant and tangent lines to a curve • computing derivatives of functions, using definitions, derivative theorems and calculator/computer methods.	Students will demonstrate conceptual understanding of derivatives, by: • showing that the slope of a tangent line is a limit • explaining how the derivative of a polynomial function can be approximated, using a sequence of secant lines • explaining how the derivative is connected to the slope of the tangent line • recognizing that f(x) = xn can be differentiated where n $\in$ R • identifying the notations f '(x), y', and $\frac{dy}{dx}$ as alternative notations for the first derivative of a function • explaining the derivative theorems for sum and difference (f $\pm$ g)'(x) = f'(x) $\pm$ g'(x) • explaining the sense of the derivative theorems for sum and difference, using practical examples. Students will demonstrate competence in the procedures associates with derivatives, by: • finding the slopes and equations of tangent lines at given points on a curve, using the definition of the derivative • estimating the numerical value of the derivative of a polynomial function at a point, using a sequence of secant lines • using the definition of the derivative to determine the derivative of f (x) = xn where n is a positive integer • differentiating polynomial functions, using the derivative theorems for sum and difference • differentiating functions that are single terms of the form xn where n is rational. Students will demonstrate problem-solving skills, by: • deriving f '(x) for polynomial functions up to the third degree, using the definition of the derivative • using the definition of the derivative to find f '(x) for f (x) = (ax + c)n where n = -1 or n = $\frac{1}{2}$.
Students are expected to understand that derivatives of more complicated functions can be found from the derivatives of simpler functions, using derivative theorems, and demonstrate this, by: • giving examples of the differences between intuitive and rigorous proofs in the context of limits, derivatives and integrals • relating the derivative of a complicated function to the derivative of simpler functions • expressing final algebraic and trigonometric answers in a variety of equivalent forms, with the form chosen to be the most suitable form for the task at hand • computing derivatives of functions, using definitions, derivative theorems and calculator/ computer methods.	Students will demonstrate conceptual understanding of derivative theorems, by: • demonstrating that the chain, power, product and quotient rules are aids to differentiate complicated functions • identifying implicit differentiation as a tool to differentiate functions where one variable is difficult to isolate • explaining the relationship between implicit differentiation and the chain rule • comparing the sum, difference, product and quotient theorems for limits and derivatives • explaining the derivation of the derivative theorems for product and quotient $(fg)'(x) = f'(x)g(x) + f(x)g'(x)$ $\left(\frac{f}{g}\right)'(x) = \frac{f'(x)g(x) - f(x)g'(x)}{[g(x)]^2}$ • explaining the derivative theorems for product and quotient, using practical examples • illustrating second, third and higher derivatives of algebraic functions • describing the second derivative geometrically.

Derivatives and Derivative Theorems

General Learner Expectations	Specific Learner Outcomes
Continued	Students will demonstrate competence in the procedures associated with derivative theorems, by: • finding the derivative of a polynomial, power, product or quotient function • applying the chain rule in combination with the product and quotient rule • using the technique of implicit differentiation • writing final answers in factored form • finding the slope and equations of tangent lines at given points on a curve • finding the second and third derivatives of functions. Students will demonstrate problem-solving skills, by: • deriving the quotient rule from the product rule • showing that equivalent forms of the derivative of a rational function can be found by using the product and the quotient rules • determining the derivative of a function expressed as a product of more than two factors • determining the second derivative of an implicitly defined function AND, in one or more of the following, by: • showing the derivative for a relation found by both implicit and explicit differentiation to be the same • finding the equations of tangent lines to the standard conics.
Students are expected to understand that trigonometric functions have derivatives, and these derivatives obey the same derivative theorems as algebraic functions, and demonstrate this, by: • identifying and giving examples of the strengths and limitations of the use of technology in the computation of limits, derivatives and integrals • giving examples of the differences between intuitive and rigorous proofs in the context of limits, derivatives and integrals • relating the derivative of a complicated function to the derivative of simpler functions • computing derivatives of functions, using definitions, derivative theorems and calculator/computer methods.	Students will demonstrate conceptual understanding of the derivatives of trigonometric functions, by: • demonstrating that the three primary trigonometric functions have derivatives at all points where the functions are defined • explaining how the derivative of a trigonometric function can be approximated, using a sequence of secant lines. Students will demonstrate competence in the procedures associated with derivatives of trigonometric functions, by: • calculating the derivatives of the three primary and three reciprocal trigonometric functions • estimating the numerical value of the derivative of a trigonometric function at a point, using a sequence of secant lines • using the power, chain, product and quotient rules to find the derivatives of more complicated trigonometric functions • using the derivative of a trigonometric function to calculate its slope at a point, and the equation of the tangent at that point. Students will demonstrate problem-solving skills, by: • using the definition of the derivative to find the derivative for the sine and cosine functions • explaining why radian measure has to be used in the calculus of trigonometric functions.

Applications of Derivatives

General Learner Expectations	Specific Learner Outcomes
Students are expected to understand that calculus is a powerful tool in determining maximum and minimum points and in sketching of curves, and demonstrate this, by: • relating the zeros of the derivative function to the critical points on the original curve • using the connections between a given problem and either a simpler or equivalent problem, or a previously solved problem, to solve the given problem • constructing mathematical models for situations in a broad range of contexts, using algebraic and trigonometric functions of a single real variable • determining the optimum values of a variable in various contexts, using the concepts of maximum and minimum values of a function • using the concept of critical values to sketch the graphs of functions, and comparing these sketches to computer-generated plots of the same functions • fitting mathematical models to situations described by data sets.	Students will demonstrate conceptual understanding of maxima and minima, by: • identifying, from a graph sketch, locations at which the first and second derivative are zero • illustrating under what conditions symmetry about the x-axis, y-axis or the origin will occur • explaining how the sign of the first derivative indicates whether or not a curve is rising or falling; and by explaining how the sign of the second derivative indicates the concavity of the graph • illustrating, by examples, that a first derivative of zero is one possible condition for a maximum or a minimum to occur • explaining circumstances wherein maximum and minimum values occur when $f'(x)$ is not zero • illustrating, by examples, that a second derivative of zero is one possible condition for an inflection point to occur • explaining the differences between local maxima and minima and absolute maxima and minima in an interval • explaining when finding a maximum value is appropriate and when finding a minimum value is appropriate. Students will demonstrate competence in the procedures associated with maxima and minima, by: • sketching the graphs of the first and second derivative of a function, given its algebraic form or its graph • using zeros and intercepts to aid in graph sketching • using the first and second derivatives to find maxima, minima and inflection points to aid in graph sketching • determining vertical, horizontal and oblique asymptotes, and domains and ranges of a function • finding intervals where the derivative is greater than zero or less than zero in order to predict where the function is increasing or decreasing • verifying whether or not a critical point is a maximum or a minimum • using a given model, in equation or graph form, to find maxima or minima that solve a problem. Students will demonstrate problem-solving skills, by: • employing a systematic calculus procedure to sketch algebraic and trigonometric functions • comparing and contrasting graphs plotted on a calculator and graphs sketched, using a systematic calculus procedure • calculating maxima and minima for such quantities as volumes, areas, perimeters and costs • illustrating the connections among geometric, economic or motion problems, the modelling equations of these problems, the resulting critical points on the graphs and their solutions, using derivatives

Applications of Derivatives

General Learner Expectations	Specific Learner Outcomes
Continued	AND ,in one or more of the following, by: • constructing a mathematical model to represent a geometric problem, and using the model to find maxima and/or minima • constructing a mathematical model to represent a problem in economics, and using the model to find maximum profits or minimum cost • constructing a mathematical model to represent a motion problem, and using the model to find maximum or minimum time or distance.
Students are expected to understand that complicated rates of change can be related to simpler rates, using the chain rule, and demonstrate this, by: • relating the derivative of a complicated function to the derivative of simpler functions • combining and modifying familiar solution procedures to form a new solution procedure to a related problem • using the connections between a given problem and either a simpler or equivalent problem, or a previously solved problem, to solve the given problem • constructing geometrical proofs at an intuitive level that generalize the concepts of slope, area, average and rate of change • expressing the connection between a derivative and the appropriate rate of change, and using this connection to relate complicated rates of change to simpler ones.	Students will demonstrate conceptual understanding of related rates, by: • illustrating how the chain rule can be used to represent the relationship between two or more rates of change • explaining the clarity that Leibnitz's notation gives to expressing related rates • illustrating the time rate of change of a function $y = f(x)$ or a relation $R(x, y) = 0$. Students will demonstrate competence in the procedures associated with related rates, by: • using the chain rule to find the derivative of a function with respect to an external variable, such as time • using Leibnitz's notation to illustrate related rates • constructing a chain rule of related rates, using appropriate variables • calculating related rates for the time derivatives of areas, volumes, surface areas and relative motion • calculating related rates of change with respect to time, given an equation that models electronic circuits or other engineering situations • using the chain rule to derive an acceleration function in terms of position, given a velocity function expressed in terms of position. Students will demonstrate problem-solving skills, by: • constructing a mathematical model to represent time rates of change of linear measures, areas, volumes, surface areas, etc. • solving related rate problems that use models containing primary trigonometric functions.

Integrals, Integral Theorems & Integral Applications

General Learner Expectations	Specific Learner Outcomes
Students are expected to understand that the operation of finding a derivative has an inverse operation of finding an antiderivative, and demonstrate this, by: • giving examples of differences between intuitive and rigorous proofs in the context of limits, derivatives and integrals • recognizing that integration can be thought of as an inverse operation to that of finding derivatives • computing definite and indefinite integrals of simple functions, using antiderivatives, integral theorems and calculator/computer methods • using the connections between a given problem and either a simpler or equivalent problem, or a previously solved problem, to solve the given problem.	Students will demonstrate conceptual understanding of antiderivatives, by: • explaining how differentiation can have an inverse operation • showing that many different functions can have the same derivative • representing, on the same grid, a family of curves that form a sequence of functions, all having the same derivative. Students will demonstrate competence in the procedures associated with antiderivatives, by: • finding the antiderivatives of polynomials, rational algebraic functions and trigonometric functions • finding the family of curves whose first derivative has been given • solving separable first order differential equations for general and specific solutions. Students will demonstrate problem-solving skills, by: • finding the antiderivatives of rational functions and polynomial powers by comparison and inspection methods • determining antiderivatives for polynomial, rational and trigonometric functions.

Integrals, Integral Theorems & Integral Applications

General Learner Expectations	Specific Learner Outcomes
Students are expected to understand that the area under a curve can be expressed as the limit of a sum of smaller rectangles, and demonstrate this, by: • identifying and giving examples of the strengths and limitations of the use of technology in the computation of limits, derivatives and integrals • giving examples of the differences between intuitive and rigorous proofs in the context of limits, derivatives and integrals • constructing geometrical proofs at an intuitive level that generalize the concepts of slope, area, average and rate of change.	Students will demonstrate conceptual understanding of area limits, by: • defining the area under a curve as a limit of the sums of the areas of rectangles • establishing the existence of upper and lower bounds for the area under a curve. Students will demonstrate competence in the procedures associated with area limits, by: • sketching the area under a curve (polynomials, rational, trigonometric) over a given interval, and approximating the area as the sum of individual rectangles Students will demonstrate problem-solving skills, by: • communicating the similarities and differences between definite integrals and areas under curves.
Students are expected to understand that the area under a curve can be related to the antiderivative of the function defining the curve; and in turn, can use the antiderivative to determine the areas under and between curves, and demonstrate this, by: • giving examples of the differences between intuitive and rigorous proofs in the context of limits, derivatives and integrals • recognizing that integration can be thought of as an inverse operation to that of finding derivatives • describing the connections between the operation of integration with the finding of areas and averages • computing definite and indefinite integrals of simple functions, using antiderivatives, integral theorems and calculator/computer methods • calculating the mean value of a function over an interval • constructing geometrical proofs at an intuitive level that generalize the concepts of slope, area, average and rate of change.	Students will demonstrate conceptual understanding of definite integrals, by: • identifying the indefinite integral $\int f(x)dx$ as a sum of an antiderivative $F(x)$ and a constant c • explaining how the definite integral between fixed limits a and b is a number whose value is $F(b) - F(a)$ • explaining the connection between the numerical values of the area and the definite integral for functions f of a constant sign and a variable sign • illustrating the following properties of integrals $\int_a^b cf(x)dx = c\int_a^b f(x)dx$ $\int_a^b [f(x) \pm g(x)]dx = \int_a^b f(x)dx \pm \int_a^b g(x)dx$ • describing the sense of the fundamental theorem of calculus • explaining how the fundamental theorem of calculus relates the area limit to the antiderivative of a function describing a curve. Students will demonstrate competence in the procedures associated with definite integrals, by: • calculating the definite integral for polynomial, rational and trigonometric functions • determining the area between a curve and the x-axis over a given domain • determining the area between a curve and the x-axis: o if $f(x)$ has a constant sign over a given interval o if $f(x)$ has a change in sign over a given interval • determining the area between curves over a given interval • determining the area between intersecting curves. Students will demonstrate problem-solving skills, by: • relating the value of the integral between $x = a$ and $x = b$ to the area between the curve and the x-axis over the interval $[a, b]$

Integrals, Integral Theorems & Integral Applications

General Learner Expectations	Specific Learner Outcomes
Continued	• using integration theorems, such as those listed below, to simplify definite integrals $\int_a^b f(x)dx = -\int_a^b f(x)dx$ $\int_a^c f(x)dx = \int_a^b f(x)dx + \int_b^c f(x)dx$ • using integral theorems to simplify more complicated integrals • illustrating the conditions necessary for a function to be differentiable, or able to be integrated.
Students are expected to understand that velocity and acceleration are the first and second derivatives of displacement with respect to time; and that once one of the three functions is known, the other two can be found by finding derivatives or antiderivatives, and demonstrate this, by: • linking displacement, velocity and acceleration of an object moving in a straight line with nonuniform velocity • calculating the displacement, velocity and acceleration of an object moving in a straight line with nonuniform velocity • calculating the mean value of a function over an interval • constructing geometrical proofs at an intuitive level that generalize the concepts of slope, area, average and rate of change.	Students will demonstrate conceptual understanding of the relationships among displacement, velocity and acceleration, by: • describing the motion of a body, using sketches of the first and second derivatives • explaining the difference between a stationary point and a turning point in the context of linear motion • illustrating the concepts of derivative and antiderivative in the context of displacement, velocity and acceleration. Students will demonstrate competence in the procedures associated with displacement, velocity and acceleration, by: • estimating an instantaneous velocity, using slopes of secant lines to represent average velocities • finding the first and second derivatives of a position function to get instantaneous velocity and instantaneous acceleration functions, where the position function is an algebraic function or a trigonometric function of time • using antiderivatives of acceleration and velocity functions to get velocity and displacement functions. Students will demonstrate problem-solving skills, by: • solving problems associated with distance, velocity and acceleration whose models are restricted to those of the forms $y'(t) = f(t)$ and $y''(t) = f(t)$ • deriving the following kinematic equations, starting from the expression a = constant: o $v = at + v_0$ o $v^2 = v_0^2 + 2ad$ o $d = ½at^2 + v_0t + d_0$ • determining the equations of velocity and acceleration in simple harmonic motion, starting from the displacement equation: o $x = A \cos (kt + c)$.

General Learner Expectations	Specific Learner Outcomes
Calculus of Exponential & Logarithmic Functions (ELECTIVE)	
Students are expected to understand that exponential and logarithmic functions have limits, derivatives and integrals that obey the same theorems as do algebraic and trigonometric functions, and demonstrate this, by: • computing limits of functions, using definitions, limit theorems and calculator/ computer methods • computing derivatives of functions, using definitions, derivative theorems and calculator/computer methods • computing definite and indefinite integrals of simple functions, using antiderivatives, integral theorems and calculator/computer methods • constructing mathematical models for situations in a broad range of contexts, using algebraic and trigonometric functions of a single real variable • fitting mathematical models to situations described by data sets.	Students will demonstrate conceptual understanding of the calculus of exponential and logarithmic functions, by: • defining exponential and logarithmic functions as inverse functions • explaining the special properties of the number *e,* together with a definition of *e* as a limit • illustrating that the derivative of an exponential or logarithmic function may be derived from the definition of the derivative • illustrating that base-*e* exponential and logarithmic functions form a convenient framework within which the calculus of similar functions in any base may be developed • illustrating how exponential and logarithmic functions may be used to model certain natural problems involving growth, decay and return to equilibrium. Students will demonstrate competence in the procedures associated with the calculus of exponential and logarithmic functions, by: • estimating the values of the limits e and e^x • approximating the slopes of $y = e^x$ and $y = ln\ x$ for some specific value of x • finding the derivative and antiderivative of the base-*e* exponential function • using limit theorems to evaluate the limits of simple exponential and logarithmic functions • finding the derivative of the natural logarithmic function • finding the derivatives of logarithmic functions having bases other than *e* • finding the derivatives and antiderivatives of exponential functions having bases other than *e* • computing $\int_a^b \frac{1}{x} dx$ and $\int_a^b e^{kx} dx$ • solving the differential equations $y' = ky$ *and* $y' = k(y - y_0)$. Students will demonstrate problem-solving skills, by: • evaluating maxima and minima of given functions involving exponential and logarithmic functions • finding areas bounded by exponential, logarithmic or reciprocal functions AND, in one or more of the following, by: • relating natural growth, natural decay and return to equilibrium to the differential equations $y' = ky$ or $y' = k(y - y_0)$ • solving natural growth and decay problems, starting from the equations $y' = ky$ or $y' = k(y - y_0)$ • fitting exponential models to observed data • calculating distance, velocity and acceleration for falling bodies, with air resistance present as part of the model • connecting the integral $\int_a^b \frac{dx}{px+q}$ with the integral $\int_a^b \frac{dx}{x}$

<table>
<tr><th>General Learner Expectations</th><th>Specific Learner Outcomes</th></tr>
<tr><td colspan="2">Numerical Methods (ELECTIVE)</td></tr>
<tr><td>Students are expected to understand that many limits, derivatives, equation roots and definite integrals can be found numerically, and demonstrate this, by:
• identifying and giving examples of the strengths and limitations of the use of technology in the computation of limits, derivatives and integrals
• computing limits of functions, using definitions, limit theorems and calculator/ computer methods
• computing derivatives of functions, using definitions, derivative theorems and calculator/computer methods
• computing definite and indefinite integrals of simple functions, using antiderivatives, integral theorems and calculator/computer methods.</td><td>Students will demonstrate conceptual understanding of the principles of numerical analysis, by:
• describing the difference between an exact solution and an approximate solution
• identifying when a particular numerical method is likely to give poor results
• explaining the difference between iterative and noniterative procedures
• explaining the basis of the Newton-Raphson procedure for determining the roots of f(x) = 0
• describing the basis of a limit, derivative, equation root or integral procedure in geometric terms
• connecting the number of subdivisions of the range of integration with the accuracy of the estimate for the integral
• showing that all numerical integration formulas are procedures that interpolate between the lower and upper Riemann sums for the integral.

Students will demonstrate competence in the procedures associated with numerical methods, by:
• estimating the value of a limit by systematic trial and error
• calculating the numerical value of the derivative at a point on a curve, whether or not there is a defining formula for the curve
• solving the equation f(x) = 0 by systematic trial and error, and by the Newton-Raphson method
• calculating the upper and lower Riemann sums for a definite integral
• calculating the value of a definite integral, using the midpoint rule
• calculating the value of a definite integral, using the trapezoidal rule
• calculating the value of a definite integral, using Simpson’s rule.

Students will demonstrate problem-solving skills, in one or more of the following:
• comparing the errors in computing definite integrals when using different procedures
• writing computer software for the computation of limits, equation roots or definite integrals
• reconstructing limit processes so that numerical evaluations can be efficient and reliable
• evaluating the reliability of a numerical procedure for finding a limit, an equation root or a definite integral.</td></tr>
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General Learner Expectations	Specific Learner Outcomes
Volumes of Revolution (ELECTIVE)	
Students are expected to understand that volumes of revolution may be considered as the limiting sum of smaller volumes and can be related to definite integrals, and demonstrate this, by: • describing the connections between the operation of integration and the finding of areas and averages • combining and modifying familiar solution procedures to form a new solution procedure to a related problem • computing definite and indefinite integrals of simple functions, using antiderivatives, integral theorems and calculator/computer methods • using the connections between a given problem and either a simpler or equivalent problem, or a previously solved problem, to solve the given problem.	Students will demonstrate conceptual understanding of volumes of revolution, by: • identifying the solid generated by the rotation of the graph of a function, either between two boundary values, or between two graphs • explaining the connection between the volume of revolution and the volume of a cylindrical disc • demonstrating how the formula for the volume of revolution by the disc method could be generated. Students will demonstrate competence in the procedures associated with volumes of revolution, by: • using the relationship $V = \pi\int_a^b [f(x)]^2 dx$ to find the volume of revolution between the boundaries of *a* and *b* for polynomial and trigonometric functions • finding the volume of revolution between two polynomial or trigonometric functions by first finding the intersection points of the graphs of the two functions. Students will demonstrate problem-solving skills, in one or both of the following, by: • deriving formulas for the volume of a cylinder, cone and sphere • revolving the graph of a function about a horizontal or vertical line, other than the *x*- or *y*-axis, and finding the resulting volume of revolution.
Applications of Calculus to Physical Sciences & Engineering (ELECTIVE)	
Students are expected to understand that most of the important equations of physics are differential equations, and calculus provides the most efficient solution method, and demonstrate this, by: • linking displacement, velocity and acceleration of an object moving in a straight line with nonuniform velocity • describing the connections between the operation of integration with the finding of areas and averages • calculating the displacement, velocity and acceleration of an object moving in a straight line with nonuniform velocity • calculating the mean value of a function over an interval • producing approximate answers to complex calculations by simplifying the models used • constructing mathematical models for situations in a broad range of contexts, using algebraic and trigonometric functions of a single real variable • fitting mathematical models to situations described by data sets.	Students will demonstrate conceptual understanding of the links among calculus, the physical sciences and engineering, by: • illustrating situations in which differential equations are required to represent problems • developing one or more differential equations in the areas of linear motion, simple harmonic motion, work, hydrostatic force, moments of inertia, radioactive decay or similar situations • showing that the concept of mean value can be applied to situations where the quantity varies with time, or where a range of values exists in a system of multiple bodies. Students will demonstrate competence in the procedures associated with the application of calculus to the physical sciences and engineering, by: • solving differential equations of type $y''(t) = f(t)$ • solving differential equations of type $y''(t) = -k^2 y$ • calculating the work done by any nonuniform force *f(x) using* $W = \int_a^b f(x)dx$ • determining root-mean-square values for sinusoidal functions. Students will demonstrate problem-solving skills, in one or more of the following, by: • determining the half-life, the decay rate, and the activity as a function of time for a radioisotope • analyzing the motion and energy in an oscillating spring system (Hooke's law) • analyzing hydrostatic forces on the surface of submerged objects • determining the moment of inertia for rigid bodies • determining the centre of mass for individual bodies and for systems of bodies • integrating Newton's second law when expressed in the form $mx''(t) = f(x)$.

General Learner Expectations	Specific Learner Outcomes
Applications of Calculus to Biological Sciences (ELECTIVE)	
Students are expected to understand that many important biological applications of calculus are connected with models involving the solution of the differential equation $f'(x) = kf(x)$, and demonstrate this, by: • combining and modifying familiar solution procedures to form a new solution procedure to a related problem • using the connections between a given problem and either a simpler or equivalent problem, or a previously solved problem, to solve the given problem • producing approximate answers to complex calculations by simplifying the models used • constructing mathematical models for situations in a broad range of contexts, using algebraic and trigonometric functions of a single real variable • fitting mathematical models to situations described by data sets.	Students will demonstrate conceptual understanding of the links between calculus and the biological sciences, by: • defining exponential and logarithmic functions as inverse functions • explaining the special properties of the number *e*, together with a definition of *e* as a limit • illustrating that the derivative of an exponential or logarithmic function may be derived from the definition of the derivative • illustrating how differential equations may be used to model certain biological problems involving growth, decay and movement across a boundary. Students will demonstrate competence in the procedures associated with the application of calculus to the biological sciences, by: • estimating the values of the limits *e* and e^x • using limit theorems to evaluate the limits of simple exponential and logarithmic functions • approximating the slopes of $y = e^x$ and $y = \ln x$ for some specific value of *x* • finding the derivative of the natural logarithmic function • finding the derivative and antiderivative of the base-*e* exponential function • using methods of guess-and-test, and comparing coefficients to solve the differential equations $y' = ky$ and $y' = k(y - y_0)$ • verifying that $y = Ae^{kx}$ satisfies the differential equation $y' = ky$. Students will demonstrate problem-solving skills, by: • relating natural growth, natural decay and return to equilibrium to the differential equations $y' = ky$ or $y' = k(y - y_0)$ • solving natural growth and decay problems starting from the equations $y' = ky$ or $y' = k(y - y_0)$ AND, either, by: • fitting differential equation models to observed biological data OR, both of the following, by: • relating growth subject to limits to the logistic equation $y' = ky(L - y)$ • solving logistic equation models, and estimating values for the parameters *k* and *L*, from experimental data.
Applications of Calculus to Business and Economics (ELECTIVE)	
Students are expected to understand that calculus may be used as a tool to analyze situations in business and economics that involve revenue, profit and cost, and demonstrate this, by: • relating the zeros of the derivative function to the critical points on the original curve • producing approximate answers to complex calculations by simplifying the models used • constructing mathematical models for situations in a broad range of contexts, using algebraic and trigonometric functions of a single real variable	Students will demonstrate conceptual understanding of the links among calculus, business and economics, by: • explaining how calculus procedures frequently arise in models used in business and economics • explaining how both maximum and minimum values are important in the making of business and economic decisions. Students will demonstrate competence in the procedures associated with the application of calculus to business and economics, by: • sketching polynomial, exponential and trigonometric functions of one variable • using a given revenue, profit or cost function to calculate and justify optimum values • finding the maximum of a revenue or profit function that is expressed as a function of price or number sold

General Learner Expectations	Specific Learner Outcomes
Continued…. • determining the optimum values of a variable in various contexts, using the concepts of maximum and minimum values of a function • using the concept of critical values to sketch the graphs of functions, and comparing these sketches to computer-generated plots of the same functions • fitting mathematical models to situations described by data sets	• finding the minimum of a cost function that is expressed as a function of price or number sold. Student will demonstrate problem-solving skills, in one or both of the following, by: • determining a revenue, profit or cost function from a problem situation that can be modelled, using polynomial functions modelling the business cycle, using trigonometric functions.
Calculus Theorems (ELECTIVE)	
Students are expected to understand that limit, derivative and integral theorems can be proved at different levels of rigor, from intuitive to analytic, and demonstrate this, by: • giving examples of the differences between intuitive and rigorous proofs in the context of limits, derivatives and integrals • calculating the mean value of a function over an interval • constructing geometrical proofs at an intuitive level that generalize the concepts of slope, area, average and rate of change.	Students will demonstrate conceptual understanding of the nature of proof in the context of limit, derivative and integral theorems, by: • proving the equivalence of the product and the quotient rule for derivatives • comparing the nature of intuitive and rigorous proofs knowing the conditions under which a theorem is true • explaining what is an example, and what is a counterexample • illustrating the intermediate value theorem, Rolle's theorem, the mean value theorem and the fundamental theorem of calculus, by examples and counterexamples, using both graphical and algebraic formulations. Students will demonstrate competence in the procedures associated with the construction of proofs of limit, derivative and integral theorems, by: • using specific functions to illustrate the equivalence of the product and quotient rules • locating an error in a given proof of a calculus theorem • verifying the mean value theorem and the fundamental theorem of calculus for specific examples. Students will demonstrate problem-solving skills, by: • composing rigorous proofs for derivative theorems, starting from the corresponding limit theorems AND, in one of the following, by: • deriving formulas for the derivatives of complicated functions, using the basic rules; e.g., functions, such as $y = \frac{f(x)g(x)}{h(x)}$ or $y = f(g(h(x)))$ • proving that a differentiable function satisfies the mean value theorem • constructing, and justifying, an iterative solution procedure for the equation $f(x) = c$, using the intermediate value theorem.

General Learner Expectations	Specific Learner Outcomes
Further Methods of Integration (ELECTIVE)	
Students are expected to understand that integration by substitution, by parts, and by partial fractions are procedures necessary when dealing with certain algebraic and trigonometric functions, and demonstrate this, by: • recognizing that integration can be thought of as an inverse operation to that of finding derivatives • expressing final algebraic and trigonometric answers in a variety of equivalent forms, with the form chosen to be the most suitable form for the task at hand • computing definite and indefinite integrals of simple functions, using antiderivatives, integral theorems and calculator/computer methods • using the connections between a given problem and either a simpler or equivalent problem, or a previously solved problem, to solve the given problem.	Students will demonstrate conceptual understanding of the methods of integration, by: • identifying integrals that cannot be evaluated, using the antiderivatives of polynomial or trigonometric functions • showing how substitutions into a definite integral change both the function to be integrated and the limits of integration • recognizing when it would be appropriate to use integration by substitution, by parts or by partial fractions. Students will demonstrate competence in the procedures associated with methods of integration, by: • using a change of variable to integrate by substitution • using a trigonometric substitution to integrate algebraic functions containing terms, such as $\sqrt{a^2 - x^2}$ • using a trigonometric identity as a first step in an integration by substitution • using integration by parts to integrate a product • writing a rational function as a sum of partial fractions • using integration by partial fractions to integrate rational algebraic functions. Students will demonstrate problem-solving skills, by: • deriving the formula for integration by parts, using the formula for the derivative of a product • combining two or more methods to integrate a rational algebraic function • adapting integration methods for antiderivatives to the evaluation of definite integrals.